Math Assignment Unit 3

Student Name: Said HASHEMI
Course: College Algebra
Date: September 22, 2025

Task 1: Quadratic Functions – Bungee Jumper Scenario

Given the height function of a bungee jumper:

$$h(t) = -0.5t^2 + v_0 t + h_0$$

Where:

- $h(t)$ is the height above the river surface (in meters),
- t is time in seconds,
- $v0$=0m/sec (initial velocity),
- $h0$=210meters (initial height).

Substituting values:

$$h(t) = -0.5t^2 + 0 \cdot t + 210 = -0.5t^2 + 210$$

This is a quadratic function in standard form $h(t)=at^2+bt+c$, with $a=-0.5$, $b=0$, and $c=210$. Since $a<0$, the parabola opens downward, indicating that the motion reaches a maximum height and then falls.

(i) Mathematical Understanding of the Concept

(a) Domain and Range of $h(t)$, and Their Physical Significance

- **Domain:** The domain represents all possible times t for which the model applies. Since time starts at $t=0$ when the jumper leaps and ends when the jumper hits the river (i.e., $h(t)=0$), we find the positive root of $h(t)=0$:

$$\begin{aligned} -0.5t^2 + 210 &= 0 \\ t^2 &= \frac{210}{0.5} = 420 \\ t &= \sqrt{420} \approx 20.49 \,\text{seconds} \end{aligned}$$

So, the domain is $[0, \sqrt{420}]$ or approximately $[0, 20.49]$ seconds.

Physical significance: The domain reflects the duration of the jump from the moment the jumper leaves the bridge until they reach the river surface. Negative time has no physical meaning here.

- **Range:** The range is the set of all possible heights during the jump. Since the parabola opens downward and the vertex is at the top, the maximum height is 210 m, and it decreases to 0 m at the end.

  Thus, the range is [0,210] meters.

  Physical significance: The range describes how high the jumper goes (maximum at 210 m) and how low they go (down to river level at 0 m). It models the vertical span of the jump.

(b) Vertex of the Function and Its Meaning

The general formula for the vertex of a parabola $h(t)=at^2+bt+c$ is at:

$$t = -\frac{b}{2a}$$

Here, $a = -0.5$, $b = 0$:

$$t = -\frac{0}{2(-0.5)} = 0$$

Then,

$$h(0) = -0.5(0)^2 + 210 = 210$$

So, the vertex is at (0,210) .

Interpretation: The vertex represents the highest point of the jump, which occurs at $t=0$. This makes sense because the jumper starts at rest ($v0 =0$) from the bridge at 210 meters. Since there is no upward initial velocity, the jumper begins falling immediately—so the maximum height is achieved at the start of the fall.

(c) Time and Maximum Height Reached

From part (b), the maximum height is 210 meters, reached at $t=0$ seconds.

Explanation: Because the initial velocity is zero, the jumper does not ascend after jumping; instead, they begin descending immediately. Thus, the maximum height is the starting height, and it occurs at the beginning of the motion.

(d) Time When the Jumper Reaches 11 Meters

We solve $h(t)=11$:

$$\begin{aligned} -0.5t^2 + 210 &= 11 \\ -0.5t^2 &= 11 - 210 = -199 \\ t^2 &= \frac{199}{0.5} = 398 \\ t &= \sqrt{398} \approx 19.95 \,\text{seconds} \end{aligned}$$

Answer: The jumper reaches 11 meters above the river at approximately 19.95 seconds.

Note: There is only one solution since the jumper is falling downward and passes through 11 m once on the way down.

(e) Height After 20 Seconds

Plug $t=20$ into the equation:

$$h(20) = -0.5(20)^2 + 210 = -0.5(400) + 210 = -200 + 210 = 10 \,\text{meters}$$

Interpretation: After 20 seconds, the jumper is 10 meters above the river. This means they are still in free fall but very close to reaching the water. Given that the total fall time is about 20.49 seconds, this is just before impact.

(f) Time When the Jumper Touches the River

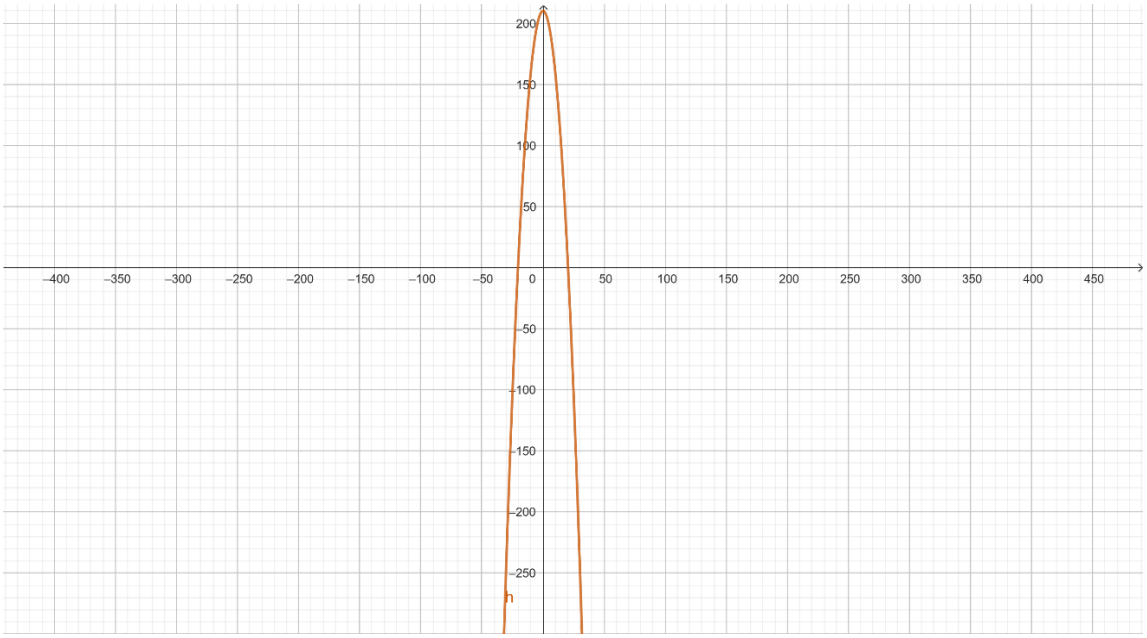
Set $h(t)=0$:

$$\begin{aligned} -0.5t^2 + 210 &= 0 \\ t^2 &= 420 \\ t &= \sqrt{420} \approx 20.49 \,\text{seconds} \end{aligned}$$

Answer: The jumper touches the river at approximately 20.49 seconds.

(ii) Graphical Understanding of the Concept

(a) Graph of $h(t)=-0.5t^2+210$



Using GeoGebra or any graphing tool, plot the function:

- Parabola opening downward.
- Vertex at $(0, 210)$
- y-intercept at $(0, 210)$
- x-intercept at $(\sqrt{420}, 0) \approx (20.49, 0)$

Graph Description: A downward-opening parabola starting at (0, 210), decreasing steadily, crossing the x-axis near 20.49.

Connection to scenario: The graph shows the jumper's descent from the bridge. The curve illustrates that the height decreases over time due to gravity, with no upward motion.

(b) Intervals Where Height Is Increasing or Decreasing

Since the vertex is at $t=0$, and the parabola opens downward:

- For $t<0$: Not physically meaningful (before jump).
- For $t>0$: The height is decreasing.

Conclusion: The height is increasing nowhere in the physical domain [0,20.49] . It is decreasing throughout the entire jump.

Reasoning: With zero initial velocity, the jumper starts falling immediately. No upward phase exists.

(c) Axis of Symmetry and Its Role

For a quadratic $h(t) = at^2 + bt + c$, axis of symmetry is:

$$t = -\frac{b}{2a} = 0$$

So, the axis of symmetry is $t = 0$.

Interpretation: This line divides the parabola into two symmetric halves. However, since the jumper starts falling at $t=0$, only the right side of the graph (for $t>0$) is relevant. The left side would represent an impossible reverse motion (going up from the bridge), so it's not physically applicable.

(d) Intercepts and Their Meaning

- **t-intercept (x-intercept):** Solve $h(t) = 0 \rightarrow t = \sqrt{420} \approx 20.49$

Represents the time when the jumper hits the river — impact time.

Represents the initial height of the jumper above the river at the moment of the jump.

- **h-intercept (y-intercept):** $h(0) = 210$

Task 2: Linear Function – Road Planning Project

Points:

- Point A: (5,7)
- Point B: (6,5)

We analyze the road connecting these points.

(i) Equation of the Road (Line Through A and B)

Use point-slope form: $y-y1=m(x-x1)$

First, compute slope m:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 7}{6 - 5} = \frac{-2}{1} = -2$$

Now use point A (5,7) :

$$y - 7 = -2(x - 5)$$
$$y - 7 = -2x + 10$$
$$y = -2x + 17$$

Equation of the road: $y=-2x+17$

(ii) Slope of the Road

As calculated: $m=-2$

Interpretation: The slope indicates a decrease of 2 units in elevation for every 1 unit increase in horizontal distance. This implies a steep downhill grade, which may require caution in traffic design.

(iii) Change in Road Elevation from A to B

Elevation change:

$$\Delta y = y_B - y_A = 5 - 7 = -2 \text{ units}$$

Horizontal change: $\Delta x=6-5=1$ unit

So, the road drops by 2 units over 1 unit horizontally.

Safety implication: A steep drop may affect vehicle control, especially in wet conditions. Engineers might consider speed limits or drainage systems.

(iv) Alternate Routes: Parallel and Perpendicular Lines

- Parallel route: Same slope $m=-2$. Example: $y=-2x+10$
- Perpendicular route: Slope is negative reciprocal of -2 , so $m\perp=1/2$

Example perpendicular line passing through A(5,7):

$$y - 7 = \frac{1}{2}(x - 5)$$
$$y = \frac{1}{2}x + 4.5$$

These alternate routes provide options for different access points or emergency exits.

(v) Visual Infrastructure Mapping

A graphical map should include:

- The original road segment between A and B.
- The full line $y=-2x+17$
- One parallel line (e.g., $y=-2x+10$)
- One perpendicular line (e.g., $y=1/2x+4.5$)
- Marked intercepts and key points

Purpose: Such maps help stakeholders visualize connectivity, gradients, and alternative paths for urban planning.

(vi) Number of Possible Parallel and Perpendicular Lines

- Parallel lines: Infinitely many can be drawn with slope -2 , differing only in y-intercept.

Perpendicular lines: Infinitely many with slope ½

Conclusion: There are infinitely many parallel and perpendicular lines possible to the proposed road.

Task 3: Linear Function – Electricity Pricing Model

Given:

- Fixed charge: $ 50
- Additional cost: $ 0.78 per unit of electricity consumed

Let:

- x = number of units of electricity consumed (in kWh)
- $C(x)$ = total monthly bill in dollars

(i) Formulate a Linear Function

$$C(x) = 0.78x + 50$$

Where:

- 0.78 is the rate per unit (slope),
- 50 is the fixed charge (y-intercept),
- $x \geq 0$ (consumption cannot be negative)

This is a linear function modeling cost as a function of usage.

(ii) Average Rate of Change and Impact on Monthly Bill

The average rate of change of a linear function is its slope, which is constant.

$$\text{Average rate of change} = \frac{\Delta C}{\Delta x} = 0.78 \,\text{dollars per unit}$$

Interpretation: For each additional unit of electricity used, the consumer pays an extra $0.78. This directly affects the monthly bill proportionally.

For example:

- If consumption increases from 100 to 200 units:

$$\Delta C = 0.78(200 - 100) = 78 \,\text{dollars}$$

So, the bill increases by $78.

Conclusion: The average rate of change determines how quickly the bill grows with consumption. Higher usage leads to higher bills at a steady rate of $0.78/unit.

References

Stitz, C., & Zeager, J. (2013). *College algebra*. Stitz Zeager Open Source Mathematics. https://stitz-zeager.com/szca07042013.pdf

Yoshiwara, K. (2020). *Modeling, functions and graphs*. American Institute of Mathematics. https://yoshiwarabooks.org/mfg/MFG.html